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OFFICE OF NAVAL RESEARCH

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Nuclear Physics Laboratory

UNIVERSITY OF NOTRE DAME

NOTRE DAME, INDIANA

NUCLEAR PHYSICS

TECHNICAL REPORT NO. 2

Relativistic Ion Optics of a Cylindrical Electrostatic Analyzer

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Abstract

The first order relativistic ion optics of a cylindrical electrostatic analyzer are developed in section I and are compared with the theories of Herzog and of Millet. The relativistic ion optics for crossed electric and magnetic fields are developed in section II. The application of these results to the Notre Dame analyzer is presented in section III.

INTRODUCTION

A 90° cylindrical electrostatic analyzer for electrons has been in operation at this laboratory for a few years and has been used in the measurement of the photodisintegration thresholds of Deuterium and Beryllium by Noyes, Van Hoomissen, Miller and Waldman (1) and in the measurement of the Ba^{137} line by S. K. Bhattacherjee (2).

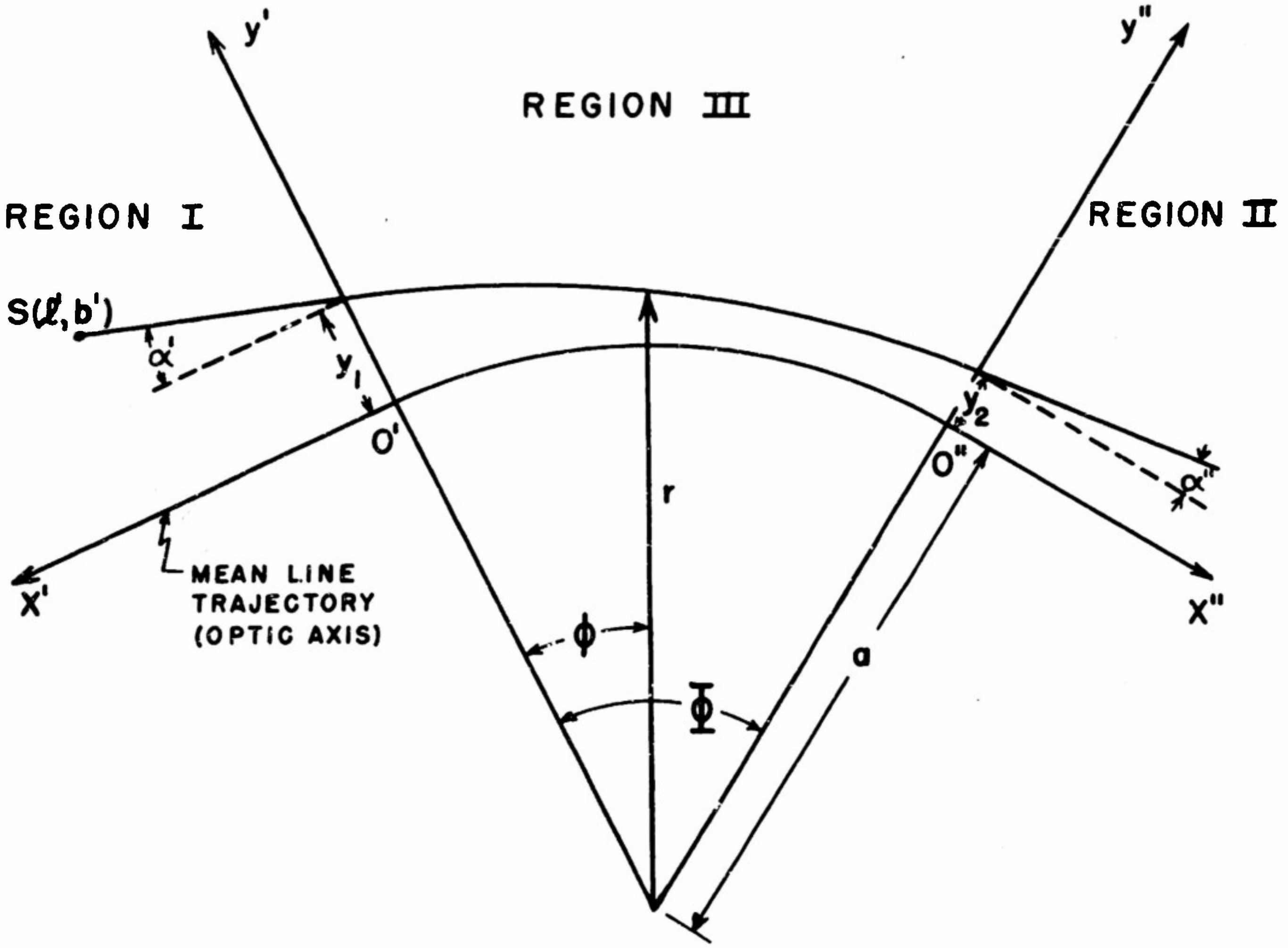
The theory of cylindrical analyzers using crossed electric and magnetic fields has been adequately reviewed by Bainbridge (3). We shall refer in particular to the works of Herzog (4) and Millet (5). Herzog considered the <u>non-relativistic</u> ion optics of cylindrical analyzers where both the source and the detector were located <u>outside</u> the region of the fields. Millet considered the <u>relativistic</u> case where both source and detector are <u>within</u> the region of the fields.

The purpose of this report is to develop in section I the first order relativistic ion optics for an electrostatic analyzer following the method and notation, where feasible, of Herzog. The results will be compared with those of Millet. Since our analyzer has a very small magnetic field (whose effect is not quite negligible) in section II we shall develop the relativistic ion optics for crossed electric and magnetic fields using the results of Millet and apply this to our analyzer in section III.

I. ELECTROSTATIC FIELD

A. Initial Conditions at Entrance to Field

Refer to Figure 1. The electrons leave the source S in a



y'
y''
REGION III
REGION I
REGION II
S(l',b')
α'
y1
y2
O'
O''
α''
r
X'
MEAN LINE
TRAJECTORY
(OPTIC AXIS)
X''
ϕ
a
Φ

FIG. 1.

field free Region I, enter the field Region III, and finally emerge into the field free Region II. In Region I the coordinate system is indicated by a single prime while in Region II it is indicated by a double prime. The coordinate x' is positive to the left from the entrance to the field O' while x" is positive to the right from the exit from the field O".

In Region I the velocity of the electrons is

$$v_I = v_0(1+\epsilon) \qquad 1$$

where $\epsilon \ll 1$ and where $v_0 = \beta_0 c$ is the velocity which an electron must have to follow the mean line trajectory $r = a$ in Region III. That is, v_0 is given by

$$\frac{m_0 v_0^2}{a} = eE_0 \qquad 2$$

where m_0 is the mean line mass* i.e., the mass of an electron which would follow the mean line trajectory, and E_0 is the field strength at $r = a$ where the potential is zero. Equation 2 can be expressed in terms of the kinetic energy T_0 and the rest energy R of the electron.

$$T_0\left(\frac{T_0 + 2R}{T_0 + R}\right)\frac{1}{a} = eE_0 \qquad 2a$$

The field strength is related to the difference of potential X between the cylindrical plates of radii r_1 and r_2

$$E_0 = \frac{1}{a}\,\frac{X}{\ln r_2/r_1}$$

*The rest mass of the electron does not appear in the following discussion.

For the usual case in which the spacing between the plates d is small compared to the arithmetic mean radius the logarithm term can be expanded. To a sufficient accuracy the geometric mean radius a and the arithmetic mean radius are equal. The final expression is

$$X \doteq \frac{T_0}{e}\left(\frac{T_0+2R}{T_0+R}\right)\frac{d}{a} \qquad 2b$$

Since the potential is zero only at $r = a$, an electron entering Region III at $r = a + y_1$ will be changed in energy due to the presence of the electric field.

$$m_{III}c^2 - m_I c^2 = -e\int_a^{a+y_1} E\,dr \doteq -y_1 E_0 e \qquad 3$$

since $y_1 \ll a$. In view of Eq. 2 this becomes

$$m_{III} \doteq m_I - m_0 \beta_0^2 \frac{y_1}{a}$$

This may be expressed in terms of the mean line mass by use of Eq. 1

$$m_{III} = m_0\left\{1 + \epsilon\frac{\beta_0^2}{1-\beta_0^2} + \cdots\right\} - m_0\beta_0^2\frac{y_1}{a}$$

or

$$m_{III} \doteq m_0\left\{1 + \epsilon\frac{\beta_0^2}{1-\beta_0^2} - \frac{\beta_0^2 y_1}{a}\right\} \qquad 4$$

Similarly the velocity at the beginning of Region III may be expressed in terms of the mean line velocity.

$$v_{III} \doteq v_0\left\{1 + \epsilon - \frac{y_1}{a}(1-\beta_0^2)\right\} \qquad 5$$

B. Equations of Motion

Within the region of the electric field the equations of motion are

$$\frac{d}{dt}(m\dot{r}) = m r \dot{\varphi}^2 - eE \qquad \text{6a}$$

$$\frac{d}{dt}(m r^2 \dot{\varphi}) = 0 \qquad \text{6b}$$

$$\frac{d}{dt}(m c^2) = -eE\dot{r} \qquad \text{6c}$$

where

$$E = \frac{X}{\ln r_2/r_1} \frac{1}{r} = \frac{A}{r} \qquad \text{7}$$

Eq. 6b can be integrated immediately.

$$m r^2 \dot{\varphi} = C \qquad \text{8}$$

Then Eq. 6 can be rewritten as functions of r, $\dot{r}$ and m

$$\frac{d}{dt}\dot{r} = \frac{C^2}{m^2 r^3} - \frac{eA}{mr} + \frac{eA}{mc^2}\frac{\dot{r}^2}{r} = f_1(r, \dot{r}, m) \qquad \text{9a}$$

$$\frac{d}{dt} r = \dot{r} = f_2(\dot{r}) \qquad \text{9b}$$

$$\frac{d}{dt} m = -\frac{eA}{c^2}\frac{\dot{r}}{r} = f_3(r, \dot{r}) \qquad \text{9c}$$

C. Approximate Equations of Motion

Following Herzog the assumption is made that in Region III all electrons travel in nearly circular orbits. That is, the values of all variables will differ only a little from the mean line values

$$r = a + z_1 \tag{10a}$$

$$\dot{r} = 0 + z_2 \tag{10b}$$

$$m = m_0 + z_3 \tag{10c}$$

where the z's are so small that z^2 can be neglected compared to z. Then the equations of motion, Eq. 9, in terms of the z's become

$$\frac{d}{dt} z_2 = f_1 \left\{ (a + z_1), (0 + z_2), (m_0 + z_3) \right\} \tag{11a}$$

$$\frac{d}{dt} z_1 = f_2 \left\{ (0 + z_2) \right\} \tag{11b}$$

$$\frac{d}{dt} z_3 = f_3 \left\{ (a + z_1), (0 + z_2) \right\} \tag{11c}$$

These equations are now expanded in Taylor series and only terms of first order in the z's are retained.

$$\frac{d}{dt} z_2 \doteq f_1(a, 0, m_0) + z_1 \left(\frac{\partial f_1}{\partial r} \right)_0 + z_2 \left(\frac{\partial f_1}{\partial \dot{r}} \right)_0 + z_3 \left(\frac{\partial f_1}{\partial m} \right)_0 \tag{12a}$$

$$\frac{d}{dt} z_1 \doteq f_2(0) + z_2 \left(\frac{\partial f_2}{\partial \dot{r}} \right)_0 \tag{12b}$$

$$\frac{d}{dt} z_3 \doteq f_3(a, 0) + z_1 \left(\frac{\partial f_3}{\partial r} \right)_0 + z_2 \left(\frac{\partial f_3}{\partial \dot{r}} \right)_0 \tag{12c}$$

where, as usual, the subscript zero refers to the mean line trajectory. In order to evaluate these coefficients it is necessary to evaluate the constant C of Eq. 8.

$$C = m r^2 \dot{\varphi} \doteq m_{III} \, r_{III} \, v_{III}$$

which from Eqs. 4 and 5 is

$$C = m_0\left\{1+\frac{\epsilon\beta_0^2}{1-\beta_0^2}-\beta_0^2\frac{y_1}{a}\right\}a\left\{1+\frac{y_1}{a}\right\}v_0\left\{1+\epsilon-\frac{y_1}{a}(1-\beta_0^2)\right\} \tag{13}$$

$$C \doteq m_0\, a\, v_0\left[1+\frac{\epsilon}{1-\beta_0^2}\right]$$

After evaluating the coefficients of Eq. 12 the approximate equations of motion become

$$\frac{d}{dt}z_2 = \frac{2v_0^2}{a}\frac{\epsilon}{1-\beta_0^2} - \frac{2v_0^2}{a^2}z_1 - \frac{v_0^2}{m_0 a}z_3 \tag{14a}$$

$$\frac{d}{dt}z_1 = z_2 \tag{14b}$$

$$\frac{d}{dt}z_3 = -\frac{m_0\beta_0^2}{a}z_2 \tag{14c}$$

The initial conditions at edge of field where $t = 0$ are

$$\begin{aligned} z_1)_i &= y_1 \\ z_2)_i &= -v_{III}\sin\alpha' \doteq -v_0\alpha' \\ z_3)_i &= m_0\beta_0^2\left[\frac{\epsilon}{1-\beta_0^2}-\frac{y_1}{a}\right] \end{aligned} \tag{15}$$

Using the method of the LaPlace Transform the solution of Eq. 14 can be shown to be

$$z_1 = a\left[-\frac{\alpha'}{K}\sin\frac{Kv_0}{a}t + \delta\left(1-\cos\frac{Kv_0}{a}t\right)+\frac{y_1}{a}\cos\frac{Kv_0}{a}t\right] \tag{16a}$$

$$z_2 = v_0 K\left[-\frac{\alpha'}{K}\cos\frac{Kv_0}{a}t + \delta\sin\frac{Kv_0}{a}t - \frac{y_1}{a}\sin\frac{Kv_0}{a}t\right] \tag{16b}$$

$$z_3 = m_0\beta_0^2\left[\frac{\alpha'}{K}\sin\frac{Kv_0}{a}t + \delta\cos\frac{Kv_0}{a}t - \frac{y_1}{a}\cos\frac{Kv_0}{a}t\right] \tag{16c}$$

where $\quad K^2 = 2 - \beta_0^2$

and $\quad \delta = \epsilon \left(\frac{1}{1-\beta_0^2} \right)$, sometimes called the dispersion coefficient. The independent variable may be changed from t to φ by applying Eqs. 10 and 15 to Eq. 8

$$\frac{d\varphi}{dt} = \frac{C}{m r^2} \doteq \frac{v_0}{a} \left[1 + \frac{\epsilon}{1-\beta_0^2} - \frac{z_2}{m_0} - \frac{2 z_1}{a} \right] \qquad 17$$

Following Herzog we retain only the zero order, viz:

$$\varphi = \frac{v_0 t}{a}$$

Thus at the Region III - Region II boundary ($x'' = 0$) the solution for $\varphi = \Phi$ is

$$y_2 = (z_1)_{\varphi = \Phi} = a \left[-\frac{\alpha'}{K} \sin K\Phi + \delta (1 - \cos K\Phi) + \frac{y_1}{a} \cos K\Phi \right] \qquad 18$$

The angle of emergence α'' is given by

$$\alpha'' \doteq \left(\frac{\dot{z}_1}{v_0} \right)_{\varphi = \Phi} = -\alpha' \cos K\Phi + \delta K \sin K\Phi - \frac{y_1}{a} \sin K\Phi \qquad 19$$

It should be noted that these equations are of the same form as the non-relativistic solution of Herzog* except for the revised definitions of δ and K . Millet obtains a similar relativistic solution for a pure electrostatic field (Millet's $g_0 = 1$). After expressing Millet's solution in our notation his Eq. 16 is

$$z_1 = -\frac{a \alpha'}{K} \sin K\varphi \qquad (M16)$$

*Equations 22a, and b of Herzog and 63 and 64 of Bainbridge.

subject to the initial conditions that $y_1 = 0$ and $\delta = 0$.
If one solves Millet's equation of motion subject to the conditions that $y_1 \neq 0$ but $\delta = 0$ one obtains (in our notation)

$$Z_1 = a\left[-\frac{\alpha'}{\kappa}\sin\kappa\varphi + \frac{y_1}{a}\cos\kappa\varphi\right] \qquad \text{(M16')}$$

D. Ion Optics

Since the solution of the relativistic problem is of the same form as Herzog's non-relativistic solution one can develop the focussing conditions exactly as he did. The equation of the trajectory in Region II is

$$y'' = y_2 + \alpha'' x''$$

or

$$y'' = a\left[-\frac{\alpha'}{\kappa}\sin\kappa\Phi + \delta(1-\cos\kappa\Phi) + \frac{y_1}{a}\cos\kappa\Phi\right] + x''\left[-\alpha'\cos\kappa\Phi + \delta\kappa\sin\kappa\Phi - \frac{y_1\kappa}{a}\sin\kappa\Phi\right] \qquad 20$$

By referring to Figure 1, y_1 can be expressed in terms of α' and the coordinates (l', b') of the source S.

$$y_1 = b' - \alpha' l' \qquad 21$$

Then Eq. 20 becomes

$$y'' = \alpha'\left[-\frac{a}{\kappa}\sin\kappa\Phi - l'\cos\kappa\Phi + x''\left(-\cos\kappa\Phi + \frac{\kappa l'}{a}\sin\kappa\Phi\right)\right] + \delta\left[a(1-\cos\kappa\Phi) + x''\kappa\sin\kappa\Phi\right] - b'\left[\frac{x''\kappa}{a}\sin\kappa\Phi - \cos\kappa\Phi\right] \qquad 22$$

For some value of $x'' = \ell''$ Eq. 22 can be made independent of α' by setting the coefficient of α' equal to zero.

$$x'' = \ell'' = \frac{\ell' \cos \chi\Phi + \frac{a}{\chi} \sin \chi\Phi}{\frac{\ell'\chi}{a} \sin \chi\Phi - \cos \chi\Phi} \tag{23}$$

At this value of x" all trajectories (for fixed δ and fixed b') intersect to form an image. The value of y'' at $x'' = \ell''$ will be called b''. Eq. 23 can be manipulated into the standard Newtonian lens equation.

$$(\ell' - g)(\ell'' - g) = f^2 \tag{24}$$

where

$$f = \frac{a}{\chi} \frac{1}{\sin \chi\Phi} \quad = \quad \text{focal length}$$

$$g = f \cos \chi\Phi \quad = \quad \text{coordinates of focal points.}$$

If one defines the magnification (lateral) in the usual optical sense

$$M = -\frac{\ell'' - g}{f} = -\frac{f}{(\ell' - g)} \tag{25}$$

then Eq. 22 can be expressed in a more convenient form.

$$b'' = \delta(1 - M)a + b'M \tag{26}$$

If a source of mean line energy ($\delta = 0$) is located at $(\ell', 0)$ the image is located at $(\ell'', 0)$. If an extended source (again $\delta = 0$)

of width b' is located in the object plane the image is of width $|M|\,b'$ (M is negative for a real image). A point source located at $(\ell', 0)$ with $\delta \neq 0$ will be imaged at (ℓ'', b'') where b'' is given by

$$b'' = \delta a (1 - M) \qquad 27$$

If this is an extended source of width b' the size of the image is given by $|M|\,b'$ and its lateral displacement is b''.

E. Dispersion

The quantity b'' is related to the velocity dispersion D_v defined as

$$D_v = \frac{b''}{\left(\frac{dv}{v_0}\right)}$$

where dv is the velocity increment. In our case this becomes

$$D_v = \frac{\delta a (1 - M)}{\epsilon}$$

or

$$D_v = \frac{(1 - M)\, a}{1 - \beta_0^2} \qquad 28$$

For the case of unit magnification ($M = -1$) this agrees with the results of Millet.

The energy dispersion coefficient D_T can be defined in terms of the fractional change in kinetic energy.

$$D_T = \frac{b''}{\left(\frac{dT}{T_0}\right)}$$

Now

$$\frac{dT}{T_0} = \frac{\beta_0^2}{1 - \beta_0^2}\left(\frac{T_0 + R}{T_0}\right)\epsilon$$

and using Eq. 27

$$D_T = \left(\frac{T_0 + R}{T_0 + 2R}\right)(1 - M)\,a \tag{29}$$

In practice the spread in energy passed by the analyzer is limited by slits of total width W' and W'' centered on the mean line trajectory and located in the object plane and image plane respectively. The fractional change in energy necessary to displace laterally the image from the center of W'' completely beyond the slit W'' can be found by use of Eq. 29.

$$D_T\left(\frac{dT}{T_0}\right) = \frac{W''}{2} - \frac{MW'}{2}$$

since M is negative for a real image.

This becomes

$$\frac{dT}{T_0} = \frac{W'' - MW'}{2a(1 - M)}\left(\frac{T_0 + 2R}{T_0 + R}\right) \tag{30}$$

Frequently the slit widths are adjusted to be in the ratio of the magnification. Then

$$\frac{dT}{T_0} = \frac{W''}{a(1 - M)}\left(\frac{T_0 + 2R}{T_0 + R}\right) \tag{31}$$

It should be noted that dT is the energy increment needed to displace the image in one direction. The total spread in energy is $\pm\, dT$. The quantity $\left(\frac{dT}{T_0}\right)$ or its reciprocal is sometimes called the energy resolution.

II. MAGNETIC FIELD

Our analyzer has a small magnetic field between the plates. Only the component normal to the plane of the orbit is of interest since the component in the plane will merely tend to deflect the electrons out of the electric field. As mentioned above, Millet has solved the problem for a relativistic ion whose complete trajectory is within the confines of such crossed fields. His solution for an ion of mean line energy ($\delta = 0$) starting at O' in the field is, in our notation,

$$z_1 = -\frac{a\alpha'}{K} \sin K\varphi \qquad \text{(M16)} \quad 32$$

where K is now given by

$$K^2 = 1 + \left(\frac{a}{a_2}\right)^2 (1 - \beta_0^2) \qquad 33$$

and a_2 is the radius of curvature of the path if only the electric field acted on the ion. Note that (a/a_2) is the ratio of the force due to the electric field to the centrifugal force. This ratio is called y by Millet. It was shown above that if one assumes that the ion starts with an initial radial displacement y, the solution is

$$z_1 = a\left[-\frac{\alpha'}{K} \sin K\varphi + \frac{y_1}{a} \cos K\varphi\right] \qquad \text{(M16')} \quad 34$$

Millet further shows that an ion starting at O' in a direction along the mean line trajectory ($\alpha' = 0$) but with a velocity $v_0(1+\epsilon)$ will be displaced from the mean line trajectory a distance

$$Z_1 = a\frac{\epsilon}{1-\beta_0^2}\,\frac{1+\left(\frac{a}{a_2}\right)(1-\beta_0^2)}{1+\left(\frac{a}{a_2}\right)^2(1-\beta_0^2)}\left[1-\cos\kappa\varphi\right] \qquad \text{(M27)} \qquad 35$$

We redefine δ similar to Herzog's definition

$$\delta = \frac{1}{\kappa^2}\,\frac{\epsilon}{1-\beta_0^2}\left\{1+\left(\frac{a}{a_2}\right)(1-\beta_0^2)\right\} \qquad 36$$

which includes the previous definition (Eq. 16) for a pure electrostatic field. Then Millet's Eq. M27 can be written

$$Z_1 = \delta\, a\,(1-\cos\kappa\varphi) \qquad \text{(M27')} \qquad 37$$

The general solution for an ion entering the field with incorrect energy is given by the superposition of Eq. M16' and Eq. M27'.

$$Z_1 = a\left[-\frac{\alpha'}{\kappa}\sin\kappa\varphi + \delta(1-\cos\kappa\varphi) + \frac{z_1}{a}\cos\kappa\varphi\right] \qquad 38$$

where

$$\kappa^2 = 1 + \left(\frac{a}{a_2}\right)^2(1-\beta_0^2)$$

and

$$\delta\kappa^2 = \frac{\epsilon}{1-\beta_0^2}\left\{1+\left(\frac{a}{a_2}\right)(1-\beta_0^2)\right\}$$

Of course, the mean line conditions are no longer given by Eq. 2. Instead one has

$$\left(\frac{a}{a_2}\right) = \frac{eE_0}{\left(\frac{m_0 v_0^2}{a}\right)} \qquad 39a$$

and

$$1-\left(\frac{a}{a_2}\right) = \frac{\frac{H e v_0}{c}}{\left(\frac{m_0 v_0^2}{a}\right)} \qquad 39b$$

In this last equation the momentum can be expressed in terms of the magnetic rigidity $[H_0\,\rho_0]$ and therefore

$$1-\left(\frac{a}{a_2}\right) = \frac{Ha}{[H_0\,\rho_0]} \qquad 40$$

Clearly, the pure electrostatic case is given by

$$\left(\frac{a}{a_2}\right) = 1$$

The negative direction of H is such that if it increases v, $\left(\frac{a}{a_2}\right)$ is greater than unity.

Since Eq. 38 is of the same form as Eq. 16 or 18, all of the ion optics formulas are valid provided one interprets K and δ in the light of Eqs. 33 and 36.

III. MAGNETIC FIELD CORRECTION

In our analyzer the magnetic field is a function of φ. There is also a magnetic field, mainly that of the earth, in Region I and Region II. Figure 2a is a plot of the vertical component of the magnetic field over the entire region, while Figure 2b is a plot of the average value of the magnetic field.

Consider first the section of the analyzer (A) between 0° and 55°. This acts like a 55° analyzer with crossed fields in which E_0 is constant and H has an average value of 0.125 gauss. Electrons which have an energy such that the pure electrostatic mean line condition of Eq. 2 is satisfied will not have the mean line energy required by Eq. 39. Consequently these electrons have a non-zero ϵ and δ which can be found from Eqs. 2, 3 and 4.

$$e E_0 = \frac{m v^2}{a} = \frac{m_0}{a}\left(1 + \frac{\epsilon \beta_0^2}{1-\beta_0^2}\right) v_0^2 (1+\epsilon)^2$$

But by Eq. 39

$$e E_0 = \left(\frac{a}{a_2}\right) \frac{m_0 v_0^2}{a}$$

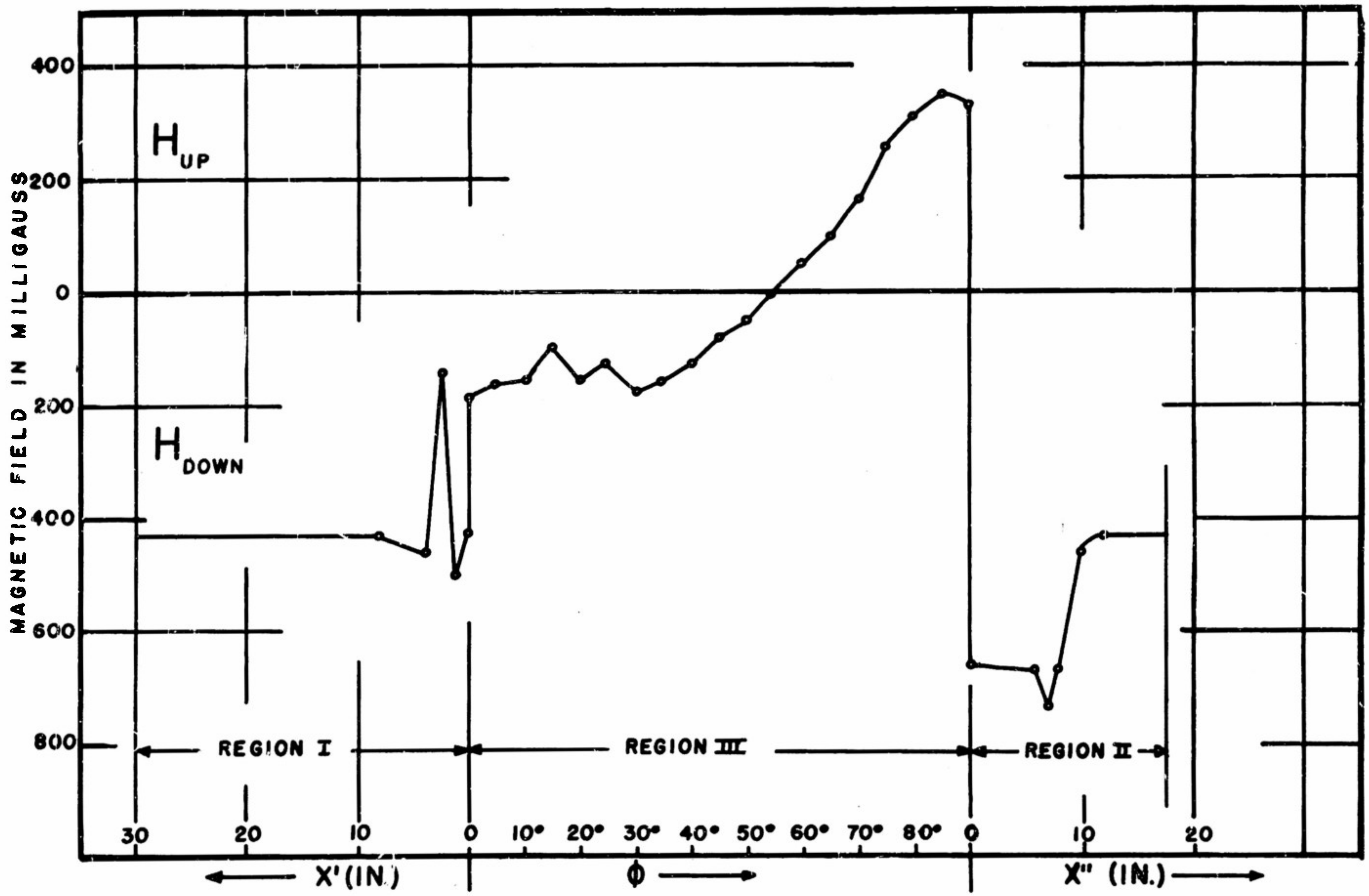


FIG. 2a. MAGNETIC FIELD DISTRIBUTION

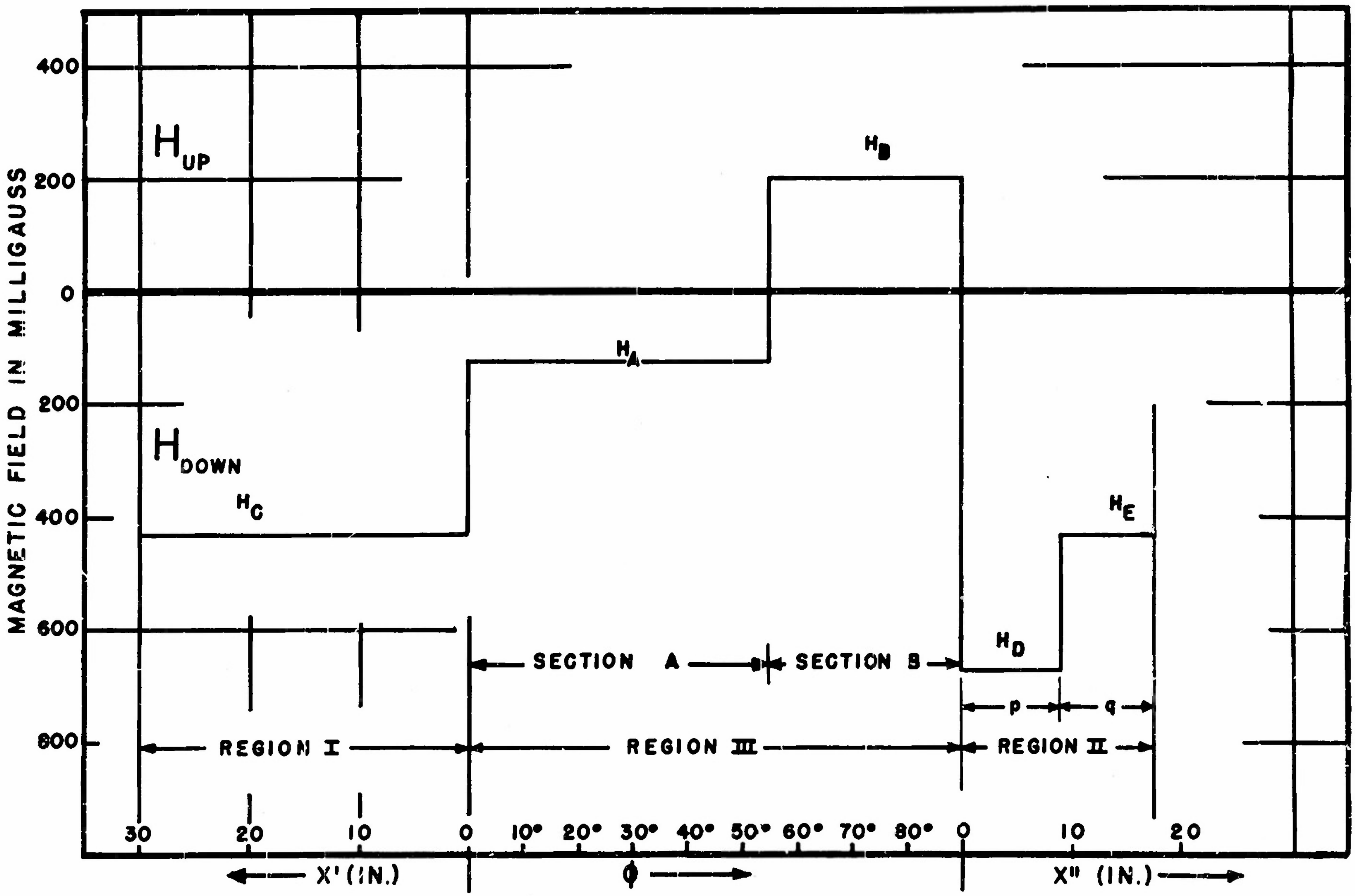


FIG. 2b. AVERAGE MAGNETIC FIELD DISTRIBUTION

and therefore

$$\frac{\epsilon}{1-\beta_o^2} = \frac{1}{2-\beta_o^2}\left(\frac{a}{a_s} - 1\right)$$

From the definition of δ (Eq. 36)

$$\delta_A \kappa_A^2 = \frac{\left(\frac{a}{a_s} - 1\right)_A}{2-\beta_o^2}\left\{1 + \left(\frac{a}{a_s}\right)\left(1-\beta_o^2\right)\right\}_A \qquad 41$$

Since H is very small (a/a_s) is very nearly unity. To a sufficient accuracy Eq. 41 may be replaced by

$$\delta_A \doteq \frac{\left(\frac{a}{a_s} - 1\right)_A}{2-\beta_o^2} \qquad 42$$

Of course, the value of β_o corresponds to the mean line energy which is not the energy of the electrons being considered. Since $(2-\beta_o^2)$ is not very energy dependent no appreciable error is made in using the value for the actual electrons.

In the section of the analyzer (B) between 55° and 90° where the average field is 0.200 gauss in the opposite direction these same electrons will have a different δ_B (actually of opposite sign since H is reversed).

In order to locate the final image we consider section A as a lens which forms an image of S at ℓ_A'' displaced from the optic axis an amount $b_A'' = \delta_A(1-M_A)a$. This image is the virtual object for the second lens B, i.e., $\ell_B' = -\ell_A''$. This object for lens B has a lateral displacement $b_B' = b_A''$ as well as a δ_B given by

$$\delta_B \doteq \frac{\left(\frac{a}{a_A} - 1\right)_B}{1 - \beta_o^2} \qquad 43$$

The image formed by lens B is at ℓ_B'' and is actually very close to the image plane for a pure 90° electrostatic analyzer. Its displacement from the mean line trajectory is

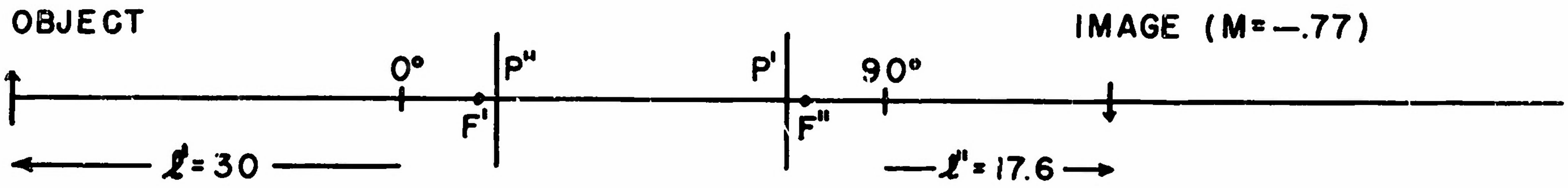
$$\begin{aligned} b_B'' &= M_B b_B' + \delta_B (1 - M_B) a \\ &= M_B \delta_A (1 - M_A) a + \delta_B (1 - M_B) a \end{aligned} \qquad 44$$

In order to clarify the above analysis we consider the lens diagram for a pure 90° electrostatic analyzer in Figure 3 and the lens diagrams for the actual analyser in Figure 4a and Figure 4b. The values of all the constants are tabulated in Table I. These have been computed for electrons of energy 2.231 Mev and for the actual source distance of 30 inches.

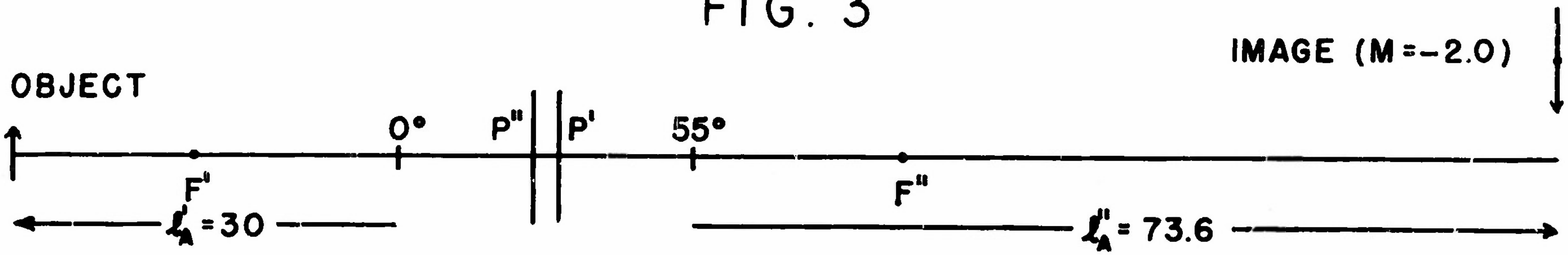
The effect of the magnetic field H_c in Region I can be appreciated very easily by realizing that the electrons follow a circular path of radius $\frac{[H_o \rho_o]}{H_c}$ between S and O' and at O' seem to be emerging from a virtual source displaced from the mean line trajectory. Elementary consideration shows that this displacement is given by

$$b' \doteq \frac{\ell'^2}{2} \frac{H_c}{[H_o \rho_o]}$$

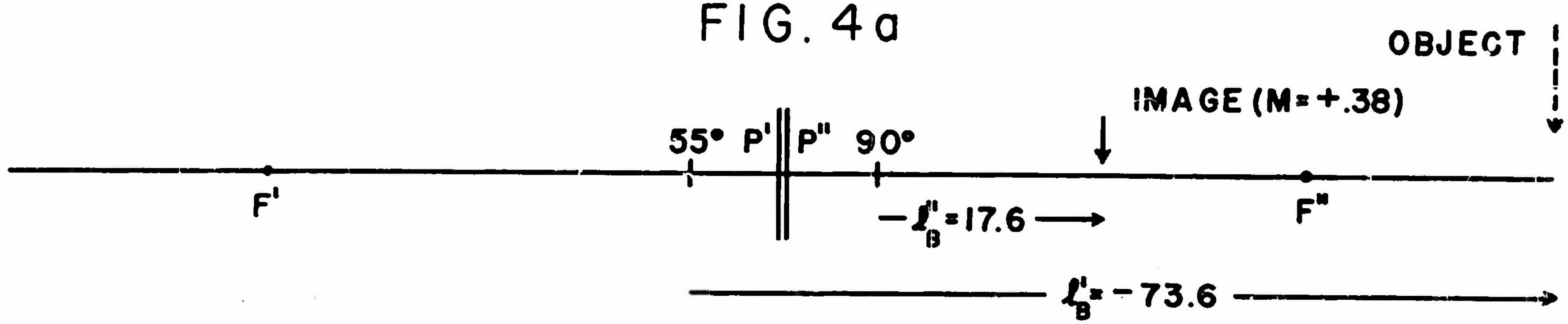
Thus, due to H_c the image of S formed by the analyzer is further displaced an amount



PURE ELECTROSTATIC
FIG. 3



SECTION A
FIG. 4a



SECTION B
FIG. 4b

F= FOCAL POINT
P= PRINCIPAL POINT

Table I

Kinetic Energy * 2.231 Mev

	Pure Electrostatic	Actual	
		Section A	Section B
$\left(\frac{a}{a_e}\right)$	1.00000	1.000849	0.999864
κ^2	1.034708	1.034767	1.034699
κ	1.017206	1.017235	1.017201
f (cm)	23.605	28.479	40.533
g (cm)	-.6379	15.947	32.956
ℓ' (cm)	30.000	30.000	-73.657
ℓ'' (cm)	17.548	73.647	17.547
M	-0.7704	-2.027	0.3802
$M_A M_B$		-0.7704	

$$b_C'' = b' M_A M_B \doteq \frac{1}{[H_o \rho_o]} \frac{l'^2}{2} H_C M$$ where $M = M_A M_B$ 45

In like fashion the field H_D in the first part (p) of Region II displaces the image an amount

$$b_D'' = \frac{1}{[H_o \rho_o]} \frac{p^2}{2} H_D \qquad 46$$

and the field H_E in the second part (q) of Region II displaces the image an amount

$$b_E'' = \frac{1}{[H_o \rho_o]} \frac{q^2}{2} H_E \qquad 47$$

The total displacement of the image is the sum of Eqs. 44, 45, 46, and 47

$$b'' = b_B'' + b_C'' + b_D'' + b_E''$$

$$= \delta_A (1 - M_A) M_B a + \delta_B (1 - M_B) a + \frac{1}{2[H_o \rho_o]} \left\{ l'^2 H_C M + p^2 H_D + q^2 H_E \right\} \qquad 48$$

In order to center this image on the axis the energy of the electrons would have to be changed an amount dT given by the energy dispersion coefficient (Eq. 29). Conversely, the determination of an unknown energy by use of Eq. 2 must be corrected by the same amount dT.

CONCLUSION

In Table II the magnetic displacement (eq. 48) has been evaluated for the cases of the photo-thresholds of deuterium and beryllium and the conversion electron energy of Ba^{137}. The energy dispersion coefficient has been evaluated from eq. 29 for an object distance ℓ' of 30 inches. The directions of the magnetic fields are such that dT is to be subtracted from the energy evaluated from eq. 2.

Table II

T_o (Mev)	b'' (inches)	D_r (inches)	$\frac{dT}{T_o}$	dT (Mev)
2.231	57×10^{-3}	35.8	1.6×10^{-3}	3.5×10^{-3}
1.664	70×10^{-3}	34.1	2.1×10^{-3}	3.4×10^{-3}
0.624	120×10^{-3}	27.5	4.4×10^{-3}	2.8×10^{-3}

Throughout the above analysis only first order terms have been retained. This imposes limitations on α' and ϵ which are not too severe in the practical case of large object distance and good resolution.

It should be noted that due to the slight energy dependence of K the object distance and the dispersion coefficients are functions of energy. Thus for a major shift in energy the position and width of the slits must be adjusted.

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